2.4 EXACT DE WITH INTEGRATING FACTORS

GIVEN $M\,dx + N\,dy = 0$

AND μ IS <u>AN</u> I.F.

SO $\mu M\,dx + \mu N\,dy = 0$ WOULD BE EXACT

IE. $\dfrac{\partial(\mu M)}{\partial y} = \dfrac{\partial(\mu N)}{\partial x}$

$$\frac{\partial \mu}{\partial y} M + \mu \frac{\partial M}{\partial y} = \frac{\partial \mu}{\partial x} N + \mu \frac{\partial N}{\partial x}$$

MORE COMPLICATED THAN ORIGINAL DE →

$$\frac{\partial \mu}{\partial y} M - \frac{\partial \mu}{\partial x} N = \mu\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$$

BUT WHAT IF WE ASSUME $\mu = \mu(x)$ (NO y IN μ)

$$\frac{\partial \mu}{\partial y} = 0$$

$$-\frac{d\mu}{dx} N = \mu\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = \mu(N_x - M_y)$$

$$\frac{d\mu}{dx} = \frac{M_y - N_x}{N}\mu$$

IF $\frac{M_y - N_x}{N}$ IS A FUNCTION OF ONLY x

$$\int \frac{d\mu}{\mu} = \int \frac{M_y - N_x}{N}\,dx$$

$$\ln|\mu| = \int \frac{M_y - N_x}{N}\,dx + C$$

$$* \quad \mu = Ce^{\int \frac{M_y - N_x}{N}dx}$$

USUALLY $C = 1$ WILL DO

ALSO IF WE ASSUME $\mu = \mu(y)$

IF $\frac{N_x - M_y}{M}$ IS A FUNCTION OF ONLY y

$$* \quad \mu = e^{\int \frac{N_x - M_y}{M}dy}$$

$$\underbrace{(y^2+2xy)}_{P}dx - \underbrace{x^2}_{Q}dy = 0$$

$$\boxed{P_y = 2y+2x} \neq \boxed{Q_x = -2x} \quad \text{NOT EXACT}$$

$$P_y - Q_x = 2y+2x- -2x$$
$$= 2y+4x$$
$$= 2(y+2x)$$

OPPOSITES

$$\boxed{\frac{Q_x - P_y}{P}} = \boxed{\frac{-2\cancel{(y+2x)}}{y\cancel{(y+2x)}}}$$

MANDATORY CHECKPOINT: FUNCTION OF y ONLY

$$\boxed{\mu = e^{\int -\frac{2}{y}dy}} = e^{-2\ln|y|} = \boxed{y^{-2}}$$

$$y^{-2}(y^2+2xy)dx - y^{-2}x^2dy = 0$$

$$\boxed{\underbrace{(1+2xy^{-1})}_{P}dx \underbrace{- x^2y^{-2}}_{Q}dy = 0}$$

$$\boxed{P_y = -2xy^{-2} = Q_x = -2xy^{-2}}$$

MANDATORY CHECKPOINT:
DE IS NOW EXACT

$$F = \int (1 + 2xy^{-1})\,dx$$

$$= x + x^2y^{-1} + C(y)$$

$$F_y = -\cancel{x^2y^{-2}} + C'(y) = -\cancel{x^2y^{-2}}$$

$$C'(y) = 0$$

MANDATORY CHECKPOINT:
ONLY CONTAINS y

$$C(y) = 0$$

$$F = x + x^2y^{-1}$$

SO $\boxed{x + x^2y^{-1} = C}$ IS A SOL'N

$$x^2y^{-1} = C - x$$

$$y^{-1} = \frac{C - x}{x^2}$$

$$\boxed{y = \frac{x^2}{C - x}}$$

2.5 SUBSTITUTION

HOMOGENEOUS 1ST ORDER ODEs

DEF'N: $f(x,y)$ IS HOMOGENEOUS OF ORDER n
IF $f(tx,ty) = t^n f(x,y)$

eg. $f(x,y) = x^2+y^2$

$$f(tx,ty) = (tx)^2+(ty)^2 = t^2x^2+t^2y^2 = t^2(x^2+y^2)$$
$$= t^2 f(x,y)$$

SO $f(x,y) = x^2+y^2$ IS HOMOGENEOUS OF ORDER 2

eg. $f(x,y) = e^{\frac{y}{x}}$

$$f(tx,ty) = e^{\frac{ty}{tx}} = e^{\frac{y}{x}} = t^0 f(x,y)$$

$f(x,y) = e^{\frac{y}{x}}$ IS HOMOGENEOUS OF ORDER 0

eg. $f(x,y) = \cos(x+y)$

$$f(tx,ty) = \cos(tx+ty) = \cos t(x+y) \neq \cancel{t\cos(x+y)}$$
$$\cancel{= t^1 f(x,y)}$$

$f(x,y)$ IS NOT HOMOGENEOUS

DEF'N: $M\,dx + N\,dy = 0$

IS A HOMOGENEOUS DE

IF M, N ARE BOTH HOMOGENEOUS OF SAME ORDER

LET $y = vx$ (WHERE $v = v(x)$)